

Pietro Menotti

# Lectures on gravitation



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## FOREWORD

The basic textbooks for this course are, in the order

R.M. Wald, General Relativity, The University of Chicago Press, Chicago and London, 1984 referred to as [Wald]

S.W. Hawking and G.F.R. Ellis, The large scale structure of space-time, Cambridge University Press, 1974 referred to as [HawkingEllis]

S. Weinberg, Gravitation and Cosmology, John Wiley and Sons, New York, 1972, referred to as [WeinbergGC]

S. Weinberg, The quantum theory of fields, Cambridge University Press, 1995, referred to as [WeinbergQFT]

Reference to other books and papers are given at the end of each section.

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