Formule utili

Gaussiana normalizzata:

$|\,A\,\rangle \xrightarrow{\text{RS}} \psi_{_A}(x) = \left(\pi\, a^2\right)^{-1/4} e^{-x^2/2a^2}$; $\qquad |\,A\,\rangle \xrightarrow{\text{RI}} \varphi_{_A}(p) = \left(\pi\,\hbar^2/a^2\right)^{-1/4} e^{-p^2a^2/2\hbar^2}$

$\overline{x^2} = \frac{1}{2}\,a^2, \qquad \overline{x^4} = \frac{3}{4}\,a^4;$ $\qquad\qquad \overline{p^2} = \hbar^2/2a^2, \qquad \overline{p^4} = 3\hbar^2/4a^4.$

Autofunzioni normalizzate dell'oscillatore armonico:

$$\psi_n(x) = \frac{1}{\sqrt{2^n\, n!}} \Big(\frac{m\,\omega}{\pi\,\hbar}\Big)^{1/4} H_n(\sqrt{m\,\omega/\hbar}\,x)\, e^{-(m\,\omega/2\,\hbar)\,x^2}$$

$$H_0(\xi) = 1\,, \quad H_1(\xi) = 2\xi\,, \quad H_2(\xi) = 4\xi^2 - 2\,.$$

Armoniche sferiche: $\qquad \displaystyle\int \big|Y_{l,m}(\theta,\,\phi)\big|^2\, d\Omega = 1\,, \qquad d\Omega = \operatorname{sen}\theta\, d\theta\, d\phi$

$$\begin{aligned}
Y_{0,0}(\theta,\phi) &= \sqrt{\frac{1}{4\pi}} \\
Y_{1,\pm1}(\theta,\,\phi) &= \sqrt{\frac{3}{8\,\pi}}\,\operatorname{sen}\theta\, e^{\pm i\,\phi} = \sqrt{\frac{3}{8\,\pi}}\,\frac{x \pm i\,y}{r} \\
Y_{1,0}(\theta,\,\phi) &= \sqrt{\frac{3}{4\,\pi}}\,\cos\theta = \sqrt{\frac{3}{4\,\pi}}\,\frac{z}{r} \\
Y_{2,\pm2}(\theta,\,\phi) &= \sqrt{\frac{15}{32\,\pi}}\,\operatorname{sen}^2\theta\, e^{\pm 2i\,\phi} = \sqrt{\frac{15}{32\,\pi}}\,\frac{(x \pm i\,y)^2}{r^2} \\
Y_{2,\pm1}(\theta,\,\phi) &= \sqrt{\frac{15}{8\,\pi}}\,\operatorname{sen}\theta\cos\theta\, e^{\pm i\,\phi} = \sqrt{\frac{15}{8\,\pi}}\,\frac{z\,(x \pm i\,y)}{r^2} \\
Y_{2,0}(\theta,\,\phi) &= \sqrt{\frac{5}{16\,\pi}}\,(1 - 3\cos^2\theta) = \sqrt{\frac{5}{16\,\pi}}\,\frac{x^2 + y^2 - 2z^2}{r^2}\,.
\end{aligned}$$

Funzioni radiali degli idrogenoidi: $\qquad \displaystyle\int_0^\infty \big|R_{n,l}(\rho)\big|^2\, \rho^2 d\rho = 1\,, \qquad \rho = Z\,r/a_{\text{B}}$

$$R_{1,0}(\rho) = 2\,e^{-\rho}; \quad R_{2,0}(\rho) = \tfrac{1}{\sqrt{2}}\,(1 - \tfrac{1}{2}\rho)\,e^{-\rho/2}; \quad R_{3,0}(\rho) = \tfrac{2}{3\sqrt{3}}\,(1 - \tfrac{2}{3}\rho + \tfrac{2}{27}\rho^2)\,e^{-\rho/3}$$

$$R_{2,1}(\rho) = \tfrac{1}{2\sqrt{6}}\,\rho\,e^{-\rho/2}; \qquad R_{3,1}(\rho) = \tfrac{8}{27\sqrt{6}}\,\rho(1 - \tfrac{1}{6}\rho)\,e^{-\rho/3}$$

$$R_{3,2}(\rho) = \tfrac{4}{81\sqrt{30}}\,\rho^2 e^{-\rho/3}$$

Si noti: $\displaystyle\int_0^\infty \left|\Big(\frac{Z}{a_{\text{B}}}\Big)^{3/2} R_{n,l}(Zr/a_{\text{B}})\right|^2 r^2 dr = 1\,.$

Matrici di Pauli:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$