

Preface

This book is a collection of three articles that have stemmed from the researches leading to my *tesi di laurea* and doctoral dissertation. They have been written in the course of several years under the ideal influence of some eminent logicians that I had the chance to meet in Pisa and during my long, or not so long, visits abroad – from Amsterdam to Tübingen, passing through Stockholm and Helsinki – and they bear a common denominator in my interest for the relationship between the calculi of sequent calculus and natural deduction. This relation has always struck me as one of the most fascinating topics of Proof Theory. The numerous and various aspects of it – which may be looked at from as much numerous and various points of view – makes it in my opinion so pregnant with significance that it becomes impossible to ignore it without failing to grasp the essential meaning of many structural properties of one or the other of the two calculi.

The articles have been reworked as chapters of a homogeneous book and arranged following the line of reasoning described and accounted for in the Introduction – which also provides a sort of “philosophical background” for them – but they need not necessarily to be read in that order. I hope that they may be of some interest and help the reader throw light on certain aspects of natural deduction and sequent calculus that have sometimes been neglected or quite misunderstood.

Contents

Preface	1
1 Introduction	11
1.1. The two formalisms	13
1.1.1. The system \mathbf{G}	14
1.1.2. The system \mathbf{N}	15
1.2. Translation functions between sequent calculus and natural de- duction	16
1.2.1. Some remarks on the translation functions	19
1.3. A comparison between the formalisms: two points of view . .	22
1.3.1. Normalization and cut-elimination compared	24
1.3.2. Towards isomorphism between the rules: natural de- duction with generalized elimination rules	27
1.3.3. Towards isomorphism between the rules: highlighted sequent calculus	30
2 A failure of correspondence between normalization and cut-eli- mination	33
2.1. The formal definition of systems \mathbf{G}_\supset^K and \mathbf{N}	34
2.1.1. High-preserving properties of \mathbf{G}_\supset^K	35
2.1.2. Cut elimination based on invertibility properties	37
2.2. Normalization, traditional cut elimination, Hudelmaier's cut elimination	47
2.2.1. Normalization, traditional cut elimination	49
2.2.2. Normalization, Hudelmaier's cut elimination	50
2.3. A deeper analysis of Hudelmaier's cut elimination	52

3	A proof of strong normalization for natural deduction with generalized elimination rules	63
3.1.	The formal definition of system \mathbf{GND}_{\supset}	63
3.2.	Strong normalization results	64
3.2.1.	Configurations	66
3.2.2.	Sufficient conditions for $\mathcal{D}\blacktriangledown$	72
3.2.3.	Preliminary lemmas	83
3.2.4.	The Strong Normalization Theorem	105
4	A highlighted sequent calculus isomorphic to natural deduction	107
4.1.	The formal definition of system \mathbf{hgG}	108
4.2.	An informal assessment	109
4.2.1.	Deductive equivalence to standard sequent calculus	110
4.3.	The isomorphism between highlighted sequent calculus and natural deduction	112
4.3.1.	Translation functions between highlighted sequent calculus and natural deduction	112
4.3.2.	What guarantees isomorphism	121
4.4.	Cut elimination results and subformula property for highlighted sequent calculus	126
4.5.	Conclusive remarks: composition, standard cut rule, concatenation rules	131