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# Proof and Truth An anti-realist perspective





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#### **PREFACE**

The present work is the result of my doctoral studies, which I carried out partially at Siena University and partially at Tübingen University between fall 2007 and fall 2010.

Enrico Moriconi was one of the members of the committee of the final oral examination and after the defence he offered me the possibility of publishing the dissertation as a volume for the ETS publisher. Although with some delay, I am very glad that I could take up his offer.

In the last two years I reworked some parts of the thesis in the form of presentations and articles. Some material ended up being published. In particular, the details of the natural deduction system for dual-intuitionistic logic appeared in Studia Logica (2012a) and the interpretation of truth-conditions and assertibilty-conditions in prooftheoretic terms in Topoi (2012b). Across chapters 2 and 3, I developed an analogy between the role of open formulas in Tarski's truth definition and that of open argumentations in Prawitz's definition of validity. This is the topic of a paper that will appear in the proceedings of the conference Logica 2012 with the title "An analogy in Dummett's views on truth- and proof-conditional meaning theories" (2013). By comparing intuitionsitic logic and its dual, I ended up with a particular analysis of the notions of proof and refutation which I am going to publish in the proceeding of the III world conference on the Square of Opposition under the title "A constructive view of the square of opposition" (forth., co-authored with Michael Arndt). I wish also to mention that previous versions of chapters 4 and 6 were published respectively as "Truth: an anti-realist adequacy condition" (2010b) and "Refutation: a proof-theoretic account" (2010a).

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Thanks to commentators, journal referees, colleagues and friends I realised how much could have been improved.

Furthermore, during the Summer term of 2012 I had the possibility of teaching a course titled "Constructive Semantics" at Siena university. This helped me a lot in getting a more firm grasp on some fundamental aspects of Dummett-Prawitz's style proof-theoretic semantics. I thank the brave students who dared follow me until the very last lesson and Gabriele Usberti for the very pleasant hospitality.

As a result of all this, now that I had to finalize the material for the publication, I had to fight a strong feeling of unease. Although I am still convinced of the fundamental ideas presented in the book, I am quite discontent with the way in which part of the material is presented.

The temptation of undertaking a serious revision of the whole thesis was strong. But I soon realised that a radical revision would have simply meant to write a new book. For this reason I tried to limit myself to essential changes.

Compared to the version I submitted to the universities libraries, the number of chapters is increased, due to the splitting of the former first and third chapter into two chapters each. With – I fear – a quite limited success, I also tried to improve the rather poor English standards.

I would recommend anyone who is interested in some particular topic to consult the mentioned articles rather than to find her or his way through the present hundred and half pages. However, I hope that at least for some reader this volume may offer a few suggestions for harmonizing in a systematic philosophical picture the different questions underlying anti-realism and its proof-theoretic semantics.

I wish to express my gratitude to Gabriele Usberti, Peter Schroeder-Heister, Enrico Moriconi and Heinrich Wansing.

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